

# **S&P Managed Risk 2.0 Index Series** *Methodology*

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# Introduction

Each index in the S&P Managed Risk 2.0 Index Series is composed of an underlying equity index, underlying fixed income index, and an underlying cash equivalent index. Each index is designed to simulate a dynamic portfolio that allocates among the underlying equity index, fixed income index, and cash equivalent index, based on the realized volatilities in the underlying equity and fixed income indices. The exposure to the underlying equity and fixed income indices is hedged with a synthetic put position to further reduce downside risk.

This methodology was created by S&P Dow Jones Indices to achieve the aforementioned objective of measuring the underlying interest of each index governed by this methodology document. Any changes to or deviations from this methodology are made in the sole judgment and discretion of S&P Dow Jones Indices so that the index continues to achieve its objective.

## **Collaboration**

The S&P Managed Risk 2.0 Index Series is generated and published under agreements between S&P Dow Jones Indices and Milliman Financial Risk Management LLC.

## **Highlights**

Each S&P Managed Risk 2.0 Index allocates to the underlying equity index and fixed income index based on the realized volatilities of the underlying equity and fixed income indices.

The target allocations to the underlying indices are calculated to maintain volatility within a fixed threshold of the target volatility level.

Each index hedges its downside risk by replicating a constant maturity put option on the volatility managed portfolio. To remove dependency on the term structure of interest rates, the strike of the option is expressed in “forward moneyness” terms. This forward strike is determined each day from a moving average of a multiple of the index level.

*Please refer to Appendix E for a full list of index parameters.*

# Index Construction

## Approaches

Each S&P Managed Risk 2.0 Index allocates to an underlying equity index and an underlying fixed income index based on the realized volatilities in the underlying indices.

To calculate the target allocation to the underlying equity and fixed income indices, the S&P Managed Risk 2.0 Index:

1. Uses an exponentially weighted risk control model to calculate the realized variance of both the underlying equity and fixed income indices, and the realized covariance between the two underlying indices.
2. Uses Black Scholes theory to calculate synthetic self-financing put option premiums and hedge allocations.
3. Adjusts the allocation weights of the underlying equity and fixed income indices in order to satisfy the portfolio target volatility and position constraints. The allocation is chosen to maximize the equity allocation. If multiple allocations are found, the allocation that also maximizes the fixed income allocation is selected.

The index hedges its downside risk by replicating a constant maturity put option on the index. The forward strike level of the put is an asymmetric exponentially weighted moving average of a multiple of the index level. The net hedge allocation is calculated per the Black Scholes model, replicating a put option with a short position in the respective S&P Managed Risk 2.0 Index and a long position in the underlying fixed income index.

## Rebalancing

The indices are rebalanced daily.

## Parameters

Parameter	Description
$M$	Protection Term
$\sigma$	Target Volatility
$\epsilon$	Target Volatility Threshold
$\tau^+$	Upward Strike Mean Reversion Period
$\tau^-$	Downward Strike Mean Reversion Period
$\tau$	Yield Premium Mean Reversion Period
$k$	Strike Multiplier
$\bar{s}$	Upper Term Yield Premium
$\underline{s}$	Lower Term Yield Premium
$\delta$	Maximum Daily Allocation Change

## Variables

Variable	Description
$\Delta t$	Time between day $t$ and $t-1$ , in years
$A$	Current Index Level
$K_t$	Current Forward Strike Price
$K_{t-1}$	Prior Day Forward Strike Price
$H$	Current Black Scholes Hedge Allocation of the Replicating Put Option
$D$	Current Modified Duration of the Underlying Fixed Income Index
$tw_E$	Target Equity Weight
$tw_B$	Target Fixed Income Weight
$vmw_E$	Volatility Management Equity Weight
$vmw_B$	Volatility Management Fixed Income Weight
$w_{E,t}$	Current Equity Weight
$w_{B,t}$	Current Fixed Income Weight
$w_{E,t-3}$	Equity Weight on $t-3$
$w_{B,t-3}$	Fixed Income Weight on $t-3$
$\sigma_t^2$	Current Target Variance
$\theta$	Maximum Volatility Management Fixed Income Weight
$r_B$	Prior Day Fixed Income Yield to Maturity
$r$	Prior Day Cash Money Market Interest Rate
$s_t$	Current Average Term Yield Premium
$s_{t-1}$	Prior Day Average Term Yield Premium
$\omega$	Maximum Fixed Income Fraction
$\varphi$	Asset Allocation Change Fraction
$E$	The Underlying Equity Index Level
$B$	The Underlying Fixed Income Index Level
$C$	The Underlying Cash Equivalent Index Level

## Total Return Index Calculations

On any business day  $t$ , the total return index value is calculated as:

$$IndexTR_t = IndexTR_{t-1} * \left( w_{E,t-3} * \left( \frac{E_t}{E_{t-1}} - 1 \right) + w_{B,t-3} * \left( \frac{B_t}{B_{t-1}} - 1 \right) + (1 - w_{E,t-3} - w_{B,t-3}) * \left( \frac{C_t}{C_{t-1}} - 1 \right) \right) \quad (1)$$

where:

$IndexTR_{t-1}$  = The total return index level on  $t-1$ .

## Put Option Replication

The index is hedged by a synthetic put option overlay. Both the put's term and volatility are constant. Its forward strike price,  $K_t$ , follows an asymmetric exponentially weighted moving average of a multiple of the index level. When the strike is below (above) the target multiple of the index level,  $kA$ , its mean reverts upward (downward) within the respective reversion period. The strike is capped at the current index level,  $A$ , to ensure the self-financing ability of the put.

$$K_t = \begin{cases} K_{t-1} + \frac{\Delta t}{\tau^+} (k * A_t - K_{t-1}) \dots \text{if } \dots K_{t-1} \leq k * A_t \\ K_{t-1} + \frac{\Delta t}{\tau^-} (k * A_t - K_{t-1}) \dots \text{if } \dots k * A_t < K_{t-1} \leq A_t \\ A_t \dots \text{otherwise} \end{cases} \quad (2)$$

To avoid borrowing funds, a portion of the index is sold to finance the put option position. This requires solving the equation below:

$$P_t = V(A_t - P_t, K_t) \quad (3)$$

where:

$P_t$  = Put option premium

$V$  = Black Scholes put option price calculated as:

$$V(A_t - P_t, K_t) = K_t * N\left(\frac{1}{\sigma\sqrt{M}} \ln \frac{K_t}{A_t - P_t} + \frac{1}{2} \sigma\sqrt{M}\right) - (A_t - P_t) * N\left(\frac{1}{\sigma\sqrt{M}} \ln \frac{K_t}{A_t - P_t} - \frac{1}{2} \sigma\sqrt{M}\right) \quad (4)$$

where  $N$  is the standard cumulative normal density function.

The net hedge allocation is calculated as:

$$H_t = -\frac{K_t}{A_t} * N\left(\frac{1}{\sigma\sqrt{M}} \ln \frac{K_t}{A_t - P_t} + \frac{1}{2} \sigma\sqrt{M}\right) \quad (5)$$

Per Black Scholes theory, replicating the put option requires a short position in the respective S&P Managed Risk 2.0 Index and a long position in the underlying fixed income index, duration-adjusted to match a zero coupon bond with maturity  $M$ . The target asset weights are thus determined from the volatility managed weights.

$$tw_{E,t} = (1 + H_t) * vmw_{E,t} \quad (6a)$$

$$tw_{B,t} = (1 + H_t) * vmw_{B,t} - H_t * \frac{M}{D} \quad (6b)$$

## Portfolio Constraints

**Volatility Constraints.** Both the short- and long-term variance of the portfolio should not exceed the current target variance.

$$tw_{E,t}^2 * EquityVariance_{S,t} + tw_{B,t}^2 * FIVariance_{S,t} + 2 * tw_{E,t} * tw_{B,t} * Covariance_{S,t} \leq \sigma_t^2 \quad (7a)$$

$$tw_{E,t}^2 * EquityVariance_{L,t} + tw_{B,t}^2 * FIVariance_{L,t} + 2 * tw_{E,t} * tw_{B,t} * Covariance_{L,t} \leq \sigma_t^2 \quad (7b)$$

See Appendix B for details on setting the current target variance.

**Position Constraints.** Leveraged and short positions are avoided.

$$tw_{E,t} + tw_{B,t} \leq 1 \quad (7c)$$

$$tw_{E,t} \geq 0 \quad (7d)$$

$$tw_{B,t} \geq 0 \quad (7e)$$

## Index Weights

**Volatility Managed Weights.** The volatility managed weights are chosen to maximize the target equity weight  $tw_E$  while satisfying the portfolio constraints (7a)-(7e). In the case that there is not a unique maximum equity exposure among the variable candidates then the solution will be chosen that furthermore maximizes the target fixed income weight  $tw_B$ .

See Appendix C and D for an explicit step-by-step algorithm.

**Final Asset Weights.** Volatility management is combined with put option replication to determine the final asset weights. Additional adjustments are made to avoid underperformance due to a persistent negative term yield premium and to limit concentrated allocation shifts:

- **Term Yield Premium.** In most market cycles a fixed income asset with positive duration provides a positive term yield premium. Should a negative term yield premium occurs, a partial or entire allocation to cash is substituted for the fixed income asset in order to avoid performance erosion.

The average term yield premium is calculated as an exponentially weighted moving average of the spread between the yield to maturity and the cash money market rate.

$$s_t = \left(1 - \frac{\Delta t}{\tau}\right) s_{t-1} + \frac{\Delta t}{\tau} (r_B - r) \quad (8)$$

The fraction of the non-equity component of the index invested in the fixed income index is capped by the maximum fixed income fraction, determined from the prior day average term yield premium as:

$$\omega_t = \min\left(1, \max\left(0, \frac{s_{t-1} - \underline{s}}{s - \underline{s}}\right)\right) \quad (9)$$

- **Daily Change Cap.** The size of the daily adjustment in the final asset weights towards the target asset weights is limited by the asset allocation change fraction, calculated as:

$$\varphi_t = \min\left(1, \frac{\delta}{|tw_{E,t} - w_{E,t-1}|}, \frac{\delta}{|\min(tw_{B,t}, \omega_t * (1 - tw_{E,t})) - w_{B,t-1}|}\right) \quad (10)$$

- **Final Asset Weights.** Apply the maximum fixed income fraction and the asset allocation change fraction, as defined in (9) and (10), to the target asset weights to determine the daily final asset weights as:

$$w_{E,t} = \varphi_t * tw_{E,t} + (1 - \varphi_t) * w_{E,t-1} \quad (11a)$$

$$w_{B,t} = \varphi_t * \min(tw_{B,t}, \omega_t * (1 - tw_{E,t})) + (1 - \varphi_t) * w_{B,t-1} \quad (11b)$$



# Index Governance

## **Index Committee**

S&P Dow Jones Indices' Global Benchmarks Index Committee maintains the indices. All Committee members are full-time professional members of S&P Dow Jones Indices' staff. The Index Committee meets monthly. At each meeting, the Index Committee reviews any significant market events. In addition, the Index Committee may revise index policy for timing of rebalancings or other matters.

S&P Dow Jones Indices considers information about changes to its indices and related matters to be potentially market moving and material. Therefore, all Index Committee discussions are confidential.

*For information on Quality Assurance and Internal Reviews of Methodology, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices document and/or Fixed Income Policies & Practices document located on our Web site, [www.spdji.com](http://www.spdji.com).*

# Index Policy

## **Announcements**

Announcements of the daily index values are made after the market close each day.

## **Holiday Schedule**

Each index is calculated daily when the underlying equity index is calculated.

*A complete holiday schedule for the year is available at [www.spdji.com](http://www.spdji.com).*

## **Unexpected Exchange Closures**

For information on Unexpected Exchange Closures, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices document located on our Web site, [www.spdji.com](http://www.spdji.com).

## **Recalculation Policy**

For information on the recalculation policy, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices and Fixed Income Policies & Practices documents for the underlying equity index and bond index, respectively. These documents are located on our Web site, [www.spdji.com](http://www.spdji.com).

*For information on Calculations and Pricing Disruptions, Expert Judgment and Data Hierarchy, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices and Fixed Income Policies & Practices documents for the underlying equity index and bond index, respectively. These documents are located on our Web site, [www.spdji.com](http://www.spdji.com).*

# Index Dissemination

Index levels are available through S&P Dow Jones Indices' Web site at [www.spdji.com](http://www.spdji.com), major quote vendors (see codes below), numerous investment-oriented Web sites, and various print and electronic media.

## Tickers

Index	Bloomberg
S&P 500 Managed Risk 2.0 Index	SPXMR2
S&P 400 Managed Risk 2.0 Index	SPMMR2
S&P 600 Managed Risk 2.0 Index	SPSMR2
S&P Emerging Plus LargeMidCap Managed Risk 2.0 Index	SPRKEMR2
S&P EPAC Ex. Korea LargeMidCap Managed Risk 2.0 Index	SPRECMR2

## FTP

Daily index levels and data are available via FTP subscription.

For product information, please contact S&P Dow Jones Indices, [www.spdji.com/contact-us](http://www.spdji.com/contact-us).

## Web site

For further information, please refer to S&P Dow Jones Indices' Web site at [www.spdji.com](http://www.spdji.com).

# Appendix A – Calculating Exponentially Weighted Variance and Covariance

On any business day  $t$ , the index calculates the realized short-term and long-term variances and covariances of the underlying equity and fixed income indices. The calculations are based on exponentially weighted moving averages.

The short-term equity variance measure at time  $t$ :

$$EquityVariance_{S,t} = \begin{cases} \lambda_S * EquityVariance_{S,t-1} + (1 - \lambda_S) * \left[ \ln\left(\frac{E_t}{E_{t-n}}\right) \right]^2 & \dots \text{if } \dots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{S,i,m}}{WF_S} * \left[ \ln\left(\frac{E_i}{E_{i-n}}\right) \right]^2 & \dots \text{if } \dots t = T_0 \end{cases} \quad (A.1a)$$

The long-term equity variance measure at time  $t$ :

$$EquityVariance_{L,t} = \begin{cases} \lambda_L * EquityVariance_{L,t-1} + (1 - \lambda_L) * \left[ \ln\left(\frac{E_t}{E_{t-n}}\right) \right]^2 & \dots \text{if } \dots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{L,i,m}}{WF_L} * \left[ \ln\left(\frac{E_i}{E_{i-n}}\right) \right]^2 & \dots \text{if } \dots t = T_0 \end{cases} \quad (A.1b)$$

The short-term fixed income variance measure at time  $t$ :

$$FIVariance_{S,t} = \begin{cases} \lambda_S * FIVariance_{S,t-1} + (1 - \lambda_S) * \left[ \ln\left(\frac{B_t}{B_{t-n}}\right) \right]^2 & \dots \text{if } \dots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{S,i,m}}{WF_S} * \left[ \ln\left(\frac{B_i}{B_{i-n}}\right) \right]^2 & \dots \text{if } \dots t = T_0 \end{cases} \quad (A.2a)$$

The long-term fixed income variance measure at time  $t$ :

$$FIVariance_{L,t} = \begin{cases} \lambda_L * FIVariance_{L,t-1} + (1 - \lambda_L) * \left[ \ln\left(\frac{B_t}{B_{t-n}}\right) \right]^2 & \dots \text{if } \dots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{L,i,m}}{WF_L} * \left[ \ln\left(\frac{B_i}{B_{i-n}}\right) \right]^2 & \dots \text{if } \dots t = T_0 \end{cases} \quad (A.2b)$$

The short-term covariance measure at time  $t$ :

$$Covariance_{S,t} = \begin{cases} \lambda_S * Covariance_{S,t-1} + (1 - \lambda_S) * \left[ \ln\left(\frac{E_t}{E_{t-n}}\right) \right] \left[ \ln\left(\frac{B_t}{B_{t-n}}\right) \right] \dots \text{if } \dots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{S,i,m}}{WF_S} * \left[ \ln\left(\frac{E_t}{E_{t-n}}\right) \right] \left[ \ln\left(\frac{B_i}{B_{i-n}}\right) \right] \dots \text{if } \dots t = T_0 \end{cases} \quad (A.3a)$$

The long-term covariance measure at time  $t$ :

$$Covariance_{L,t} = \begin{cases} \lambda_L * Covariance_{L,t-1} + (1 - \lambda_L) * \left[ \ln\left(\frac{E_t}{E_{t-n}}\right) \right] \left[ \ln\left(\frac{B_t}{B_{t-n}}\right) \right] \dots \text{if } \dots t > T_0 \\ \sum_{i=m+1}^{T_0} \frac{\alpha_{L,i,m}}{WF_L} * \left[ \ln\left(\frac{E_t}{E_{t-n}}\right) \right] \left[ \ln\left(\frac{B_i}{B_{i-n}}\right) \right] \dots \text{if } \dots t = T_0 \end{cases} \quad (A.3b)$$

where:

$E_t$  = The underlying equity index level at time  $t$ .

$B_t$  = The underlying fixed income index level at time  $t$ .

$T_0$  = The start date of the S&P Managed Risk 2.0 Index.

$n$  = The number of days in the return calculation.  $n = 1$  as daily returns are used to calculate realized volatility.

$m$  = The  $m^{\text{th}}$  trading date prior to  $T_0$ .

$N$  = The number of trading days observed for calculating the initial variance as of the start date of the index.  $N = 60$ .

$\lambda_S$  = The short-term decay factor used for exponential weighting. The decay factor is a number greater than zero and less than one that determines the weight of each daily return in the calculation of historical variance.  $\lambda_S = 0.94$ .

$\lambda_L$  = The long-term decay factor used for exponential weighting. The decay factor is a number greater than zero and less than one that determines the weight of each daily return in the calculation of historical variance.  $\lambda_L = 0.97$ .

$\alpha_{S,m,i}$  = Weight of date  $t$  in the short-term volatility calculation, as determined based on the following formulae:

$$\alpha_{S,t} = (1 - \lambda_S) * \lambda_S^{N+m-i} \quad (A.4a)$$

$$WF_S = \sum_{i=m+1}^{T_0} \alpha_{S,i,m} \quad (A.4b)$$

$\alpha_{L,m,i}$  = Weight of date  $t$  in the long-term volatility calculation, as determined based on the following formulae:

$$\alpha_{L,t} = (1 - \lambda_L) * \lambda_L^{N+m-i} \quad (A.5a)$$

$$WF_L = \sum_{i=m+1}^{T_0} \alpha_{L,i,m} \quad (A.5b)$$

## Appendix B – Setting the Current Target Variance

Volatility of the index is allowed to deviate by a small threshold amount from the target level in order to reduce the occurrence of minor asset weight adjustments.

$$\sigma - \varepsilon \leq \sigma_t \leq \sigma + \varepsilon \quad (\text{B.1})$$

Neither the short- nor long-term volatility of the underlying portfolio should substantially exceed the target level. The current target variance is defined as:

$$\sigma_t^2 = \min\left((\sigma + \varepsilon)^2, \max\left((\sigma - \varepsilon)^2, \text{Variance}_{S,t}, \text{Variance}_{L,t}\right)\right) \quad (\text{B.2})$$

where:

$\text{Variance}_{S,t}$  = Short-term variance of the portfolio calculated as:

$$\text{Variance}_{S,t} = \frac{w_{E,t-1}^2 * \text{EquityVariance}_{S,t} + (w_{B,t-1} + H \frac{M}{D})^2 * \text{FIVariance}_{S,t} + 2 * w_{E,t-1} * (w_{B,t-1} + H \frac{M}{D}) * \text{Covariance}_{S,t}}{(1 + H)^2} \quad (\text{B.3a})$$

$\text{Variance}_{L,t}$  = Long-term variance of the portfolio calculated as:

$$\text{Variance}_{L,t} = \frac{w_{E,t-1}^2 * \text{EquityVariance}_{L,t} + (w_{B,t-1} + H \frac{M}{D})^2 * \text{FIVariance}_{L,t} + 2 * w_{E,t-1} * (w_{B,t-1} + H \frac{M}{D}) * \text{Covariance}_{L,t}}{(1 + H)^2} \quad (\text{B.3b})$$

## Appendix C – Relaxing the Leverage Constraints

Zero volatility managed weights imply a long fixed income weight of:

$$tw_{B,t} = -H_t * \frac{M}{D}$$

This may exceed one if  $M > D$  and  $H \approx -1$ . Leveraged and short positions are avoided, but in this case a small amount of leverage is permitted in the volatility managed fixed income weight in order to include zero volatility in the solution domain. This requires a small relaxation of the constraint (7c) by replacing it with:

$$\theta * tw_{E,t} + tw_{B,t} \leq \theta \tag{C.1}$$

where:

$$\theta = \max\left(-H \frac{M}{D}, 1\right) \tag{C.2}$$

Note that this is merely a technical consideration since the final fixed income weight  $w_{B,t}$  is capped and thus will not result in a leveraged portfolio holding.

# Appendix D – Calculating Volatility Managed Weights

The volatility management weights are the solution to a quadratic program. This Appendix provides a step-by-step algorithm to find the solution.

As the solution to a convex quadratic program, the volatility management weights are found on the boundary implied by the constraints (7a), (7b), (B.1), (7d) and (7e).

## Candidate Boundary Solution Points

**Maximum Leverage Boundary.** The first boundary considered is the maximum leverage boundary corresponding to constraint (B.1):

$$\theta * tw_{E,t} + tw_{B,t} = \theta$$

There are up to four points on this boundary matching the current target variance:

$$vmw_{E,t} = \frac{(\theta * FIVariance_{S,t} - Covariance_{S,t}) * \frac{\theta + H \frac{M}{D}}{1+H}}{EquityVariance_{S,t} - 2\theta * Covariance_{S,t} + \theta^2 * FIVariance_{S,t}}$$

$$\pm \frac{\sqrt{(EquityVariance_{S,t} - 2\theta * Covariance_{S,t} + \theta^2 * FIVariance_{S,t}) * \sigma_t^2 - (EquityVariance_{S,t} * FIVariance_{S,t} - Covariance_{S,t}^2) \left( \frac{\theta + H \frac{M}{D}}{1+H} \right)^2}}{EquityVariance_{S,t} - 2\theta * Covariance_{S,t} + \theta^2 * FIVariance_{S,t}} \quad (D.1)$$

$$vmw_{E,t} = \frac{(\theta * FIVariance_{L,t} - Covariance_{L,t}) * \frac{\theta + H \frac{M}{D}}{1+H}}{EquityVariance_{L,t} - 2\theta * Covariance_{L,t} + \theta^2 * FIVariance_{L,t}}$$

$$\pm \frac{\sqrt{(EquityVariance_{L,t} - 2\theta * Covariance_{L,t} + \theta^2 * FIVariance_{L,t}) * \sigma_t^2 - (EquityVariance_{L,t} * FIVariance_{L,t} - Covariance_{L,t}^2) \left( \frac{\theta + H \frac{M}{D}}{1+H} \right)^2}}{EquityVariance_{L,t} - 2\theta * Covariance_{L,t} + \theta^2 * FIVariance_{L,t}} \quad (D.2)$$

And corresponding volatility managed fixed income weight:

$$vmw_{B,t} = \frac{\theta + H \frac{M}{D}}{1+H} - \theta * vmw_{E,t} \quad (D.3)$$



**Minimum Fixed Income Boundary.** The second boundary considered is the Minimum fixed income boundary corresponding to constraint (7e):

$$tw_{B,t} = 0$$

There are up to four points on this boundary matching the current target variance:

$$vmw_{E,t} = \frac{-\frac{H}{1+H} \frac{M}{D} Covariance_{S,t} \pm \sqrt{\sigma_t^2 EquityVariance_{S,t} - \left(\frac{H}{1+H} \frac{M}{D}\right)^2 (EquityVariance_{S,t} * FIVariance_{S,t} - Covariance_{S,t}^2)}}{EquityVariance_{S,t}} \quad (D.4)$$

$$vmw_{E,t} = \frac{-\frac{H}{1+H} \frac{M}{D} Covariance_{L,t} \pm \sqrt{\sigma_t^2 EquityVariance_{L,t} - \left(\frac{H}{1+H} \frac{M}{D}\right)^2 (EquityVariance_{L,t} * FIVariance_{L,t} - Covariance_{L,t}^2)}}{EquityVariance_{L,t}} \quad (D.5)$$

And corresponding volatility managed fixed income weight:

$$vmw_{B,t} = \frac{H}{1+H} \frac{M}{D} \quad (D.6)$$

**Target Variance Boundary.** The maximum target equity weight obtained while matching the current target variance must be one of the following four points.

The short-term volatility boundary corresponding to constraint (7a)

$$tw_{E,t}^2 * EquityVariance_{S,t} + tw_{B,t}^2 * FIVariance_{S,t} + 2 * tw_{E,t} * tw_{B,t} * Covariance_{S,t} = \sigma_t^2$$

has one solution:

$$vmw_{E,t} = \sigma_t * \sqrt{\frac{FIVariance_{S,t}}{EquityVariance_{S,t} * FIVariance_{S,t} - Covariance_{S,t}^2}} \quad (D.7)$$

$$vmw_{B,t} = -\frac{Covariance_{S,t}}{FIVariance_{S,t}} * vmw_{E,t} \quad (D.8)$$

The long-term volatility boundary corresponding to constraint (7b)

$$tw_{E,t}^2 * EquityVariance_{L,t} + tw_{B,t}^2 * FIVariance_{L,t} + 2 * tw_{E,t} * tw_{B,t} * Covariance_{L,t} = \sigma_t^2$$

has one solution:

$$vmw_{E,t} = \sigma_t * \sqrt{\frac{FIVariance_{L,t}}{EquityVariance_{L,t} * FIVariance_{L,t} - Covariance_{L,t}^2}} \quad (D.9)$$

$$vmw_{B,t} = -\frac{Covariance_{L,t}}{FIVariance_{L,t}} * vmw_{E,t} \quad (D.10)$$

The intersection of the short- and long-term volatility boundaries has four points:

$$vmw_{E,t} = \mu\lambda \quad (D.11)$$

$$vmw_{B,t} = (1 - \mu)\lambda \quad (D.12)$$

where:

$$\mu = \frac{FIDiff - CovDiff \pm \sqrt{CovDiff^2 - EqDiff * FIDiff}}{EqDiff - 2 * CovDiff + FIDiff} \quad (D.13)$$

$$\lambda = \frac{\sigma_t}{\sqrt{(EquityVariance_{S,t} - 2 * Covariance_{S,t} + FIVariance_{S,t}) * \mu^2 + 2 * (Covariance_{S,t} - FIVariance_{S,t}) * \mu + FIVariance_{S,t}}} \quad (D.14)$$

$$EqDiff = EquityVariance_{E,t} - EquityVariance_{L,t} \quad (D.15)$$

$$FIDiff = FIVariance_{E,t} - FIVariance_{L,t} \quad (D.16)$$

$$CovDiff = Covariance_{E,t} - Covariance_{L,t} \quad (D.17)$$

**Maximum Equity Corner.** The final boundary point is the corner at the intersection of constraints (7c) and (7e):

$$vmw_{B,t} = \frac{1}{1 + H} \quad (D.18)$$

$$vmw_{B,t} = \frac{H \frac{M}{D}}{1 + H} \quad (D.19)$$

### Selecting the Solution

The values defined in (D.1)-(D.19) are not all well-defined. Only those solutions that are both well-defined and satisfy all the constraints are considered. From these solutions, the one with the greatest equity exposure  $vmw_{E,t}$  is selected. In a situation where there is not a unique maximum equity exposure among the candidates, the solution that also maximizes the target fixed income weight  $vmw_{B,t}$  will be selected.

# Appendix E - Parameters

## General Parameters

Interest Rate:	U.S. Overnight Federal Funds Rate
Short Term Decay Factor ( $\lambda_S$ ):	0.94
Long Term Decay Factor ( $\lambda_L$ ):	0.97
Time to Maturity (Put Option), $T$ :	5 Years
Strike Multiplier ( $k$ ):	0.8
Target Volatility:	22%
Target Volatility Band:	0.50%
Upper Yield Premium:	0.25%
Lower Yield Premium:	0.00%
Maximum Daily Allocation Change:	10%
Protection Mean Reversion Period Down:	2.00
Protection Mean Reversion Period Up:	0.75
Yield Premium Mean Reversion Period:	2.50

## Index-Specific Parameters

Index	Equity	Underlying Indices	
		Fixed Income	Cash Equivalent
S&P 500 Managed Risk 2.0 Index	S&P 500	S&P U.S. Treasury Bond Current 5-Year Index	S&P U.S Treasury Bill 0-3 Month Index
S&P 400 Managed Risk 2.0 Index	S&P MidCap 400		
S&P 600 Managed Risk 2.0 Index	S&P SmallCap 600		
S&P Emerging Plus LargeMidCap Managed Risk 2.0 Index	S&P Emerging Plus LargeMidCap		
S&P EPAC Ex. Korea LargeMidCap Managed Risk 2.0 Index	S&P EPAC Ex. Korea LargeMidCap		

## Currency of Calculation

The indices are calculated in U.S. dollars.

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